Course guide
300501 - CAL-S - Calculus

Last modified: 11/07/2024

Unit in charge: Castelldefels School of Telecommunications and Aerospace Engineering
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN SATELLITE ENGINEERING (Syllabus 2024). (Compulsory subject).

Academic year: 2024 **ECTS Credits:** 6.0 **Languages:** Catalan

LECTURER

Coordinating lecturer: Definit a la infoweb de l'assignatura

Others: Definit a la infoweb de l'assignatura

TEACHING METHODOLOGY

In the theory lectures, the fundamental concepts of the subject will be introduced, and basic exercises and problem-solving techniques will be presented. In the problems sessions, exercises and problems proposed a priori by the lecturer and autonomously prepared by the students will be discussed and solved.

There will be some sessions where the students will have to bring their computer or tablet to design small Python programs that solve problems related to the subject (without using paper and pen).

LEARNING OBJECTIVES OF THE SUBJECT

At the end of the Calculus course, students must be able to:
• Operate and represent correctly real and complex numbers.
• Find the real and complex roots of a polynomial with real coefficients.
• Understand the concept of the limit of a function in a point and know some techniques to calculate them.
• Solve problems involving derivatives of functions of one variable.
• Know Taylor’s formula and its application to the local study and approximate evaluation of functions.
• Understand the concept of integral, and calculate areas of regions in R2, and volumes of some solids in R3.
• Know some techniques for the calculation of primitives.
• Identify conics and quadrics from their equations.
• Acquire skills in calculations involving curves and surfaces.
• Understand and know how to geometrically interpret the concepts of directional and partial derivatives and gradient.
• Calculate local and absolute extrema of functions of one variable and conditioned extrema of functions of several variables.
• Reading comprehension: understanding the statement of a problem related to satellite engineering to apply mathematical techniques that lead to its resolution.
• Solve mathematical problems with the help of programming languages by designing small algorithms and routines (in Python).

LEARNING OUTCOMES
Knowledge
K1. Identify the mathematical tools applied in the mechanical structures of satellites.

Skills
S1. Apply basic and advanced mathematical concepts to problems related to space engineering.
S2. Solve mathematical problems by designing models that fit the behaviour of problems related to space and satellite engineering.

Competences
C1. Develop the learning skills necessary to tackle other subjects more independently.

CONTENTS

1 .Real and Complex numbers; functions of one real variable; curves and regions in the space R2

Description:
1.1 Real and complex numbers. The Euler's formula. n-th roots.
1.2 Functions of one real variable: definition, domain, graph, examples of elemental functions. Polynomials.
1.3 Equations in two variables, curves in R2. The conic curves.
1.4 Inequations and regions in R2.

Specific objectives:
• To operate with real and complex numbers and to represent them geometrically.
• To graphically represent elemental functions of one real variable.
• To find all the roots (real and complex) of a polynomial with real coefficients.
• To identify a conic curve from its equation and to graphically represent it.
- To solve inequations with one and two variables, both analytically and geometrically.

Related activities:
• Problems resolution
• One PC session to solve problems with Python programs
• Test C1
• Mid-term exam

Full-or-part-time: 28h
Theory classes: 11h
Self study : 17h

2. Derivation of functions of one variable, and applications

Description:
2.1 Concept of derivative: analytical definition and geometrical interpretation.
2.2 Calculation of derivatives: the chain rule, implicit derivation.
2.3 Application 1: tangent and orthogonal lines to a curve in R2.
2.4 Application 2: limits and indeterminacies, l'Hôpital rule.
2.5 Application 3: approximation of functions, Taylor polynomial, Lagrange residue. Extension to Taylor powers series.
2.6 Application 4: functions analysis: increase/decrease, relative extrema, absolute extrema, optimization problems.

Specific objectives:
• To be able to derivate any explicit function of one variable.
• To find the equation of a line tangent to a curve in R2 which goes through a given point.
• To calculate limits that initially are equal to an indeterminacy through l'Hôpital rule.
• To calculate Taylor polynomials and use them to approximate functions and values with bounds on the corresponding error.
• To manipulate numerical and power series, particularly Taylor ones.
• To study the increase or decrease of a function of one variable by finding its relative and absolute extrema.
• To solve optimization problems related to real-life situations, particularly satellite engineering.

Related activities:
• Problems resolution
• One PC session to solve problems with Python programs
• Test C1
• Mid-term exam

Full-or-part-time: 28h
Theory classes: 11h
Self study : 17h

3. Integration of functions of one variable and applications

Description:
3.1 Indefinite integral: primitive of a function, immediate primitives.
3.2 Techniques for calculating primitives: almost-immediate primitives, integration by change of variable, integration by parts, integration of rational, trigonometric, and irrational functions.
3.3 Definite integral: definition, geometrical interpretation, Barrow's rule.
3.4 Applications of the definite integral: calculation of areas of planar figures and volumes of some solids of revolution.
3.5 Improper integrals.

Specific objectives:
• To calculate primitives of functions using a wide variety of integration techniques.
• To calculate areas of regions in R2 and length of curves in R2 using definite integrals.
• To identify and calculate improper integrals.

Related activities:
• Problems resolution
• One PC session to solve problems with Python programs
• Test C2
• Final term exam

Full-or-part-time: 28h
Theory classes: 11h
Self study : 17h

4. Functions of several variables. Surfaces and parameterized curves

Description:
4.1 Functions from R2 to R: domain, contours, and sections.
4.2 Definition of surface in R3. The example of quadric surfaces.
4.3 Functions from R to R2 or to R3. Parameterized trajectories, tangent vector, velocity.
4.4 Parameterization of conic curves.

Specific objectives:
• To calculate and graphically represent section curves and contours.
• To parameterize any conic curve in R2.
• To identify quadric surfaces from their equation.
• To find equations of tangent vectors and lines to a curve in R2 or R3 from the parameterization of the curve.
• To express a parameterized curve in R3 as the intersection of two surfaces.

Related activities:
• Problems resolution
• Test C2 and Programming Test
• Final term exam

Full-or-part-time: 23h
Theory classes: 8h
Self study : 15h

DIfferential calculus in Rn

Description:
5.1 Definition and calculation of directional and partial derivatives, and the gradient vector.
5.2 Application 1 of the gradient vector: directions of maximum / minimum / not increase.
5.3 Application 2 of the gradient vector: tangent plane to a surface in R3.
5.4 Differentiability of vector functions. Jacobian matrix.
5.5 Second partial derivatives. Schwarz's Theorem. Taylor polynomial for functions of two variables.
5.6 Extrema of scalar functions in regions of R2 and R3: compact regions, Weierstrass' Theorem, algorithms for searching absolute extrema (parameterization method, Lagrange multipliers).

Specific objectives:
- To calculate partial derivatives and gradient vectors.
- To find directions of maximum increase or decrease, and no variation directions, of a function in a point.
- To find the equation of a tangent plane to a surface in R3 through a point of the surface.
- To find the absolute extrema of a scalar function in a compact region of R2 or R3.

Related activities:
- Problems resolution
- One PC session to solve problems with Python programs
- Final term exam

Full-or-part-time: 43h
Theory classes: 16h
Self study : 27h

ACTIVITIES

Test C1

Description:
Written exam done during large-group hours about contents 1 and 2.

Specific objectives:
Continuous evaluation: The goal is to make students constantly follow the contents of the subject.

Full-or-part-time: 6h
Self study: 5h
Theory classes: 1h

Test C2

Description:
Written exam done during large-group hours, about contents 3 and 4.

Specific objectives:
Continuous evaluation: The goal is to make students constantly follow the contents of the subject.

Full-or-part-time: 6h
Self study: 5h
Theory classes: 1h

Mid-term exam

Description:
A written exam done during the EETAC mid-term exams week about contents 1, 2, and 3.

Specific objectives:
Continuous evaluation: The goal is to make students constantly follow the contents of the subject.

Full-or-part-time: 6h 30m
Self study: 5h
Theory classes: 1h 30m

Final-term exam

Description:
A written exam done during the EETAC final term exams week about contents 4 and 5.

Specific objectives:
Continuous evaluation: The goal is to make students constantly follow the contents of the subject.

Full-or-part-time: 6h 30m
Self study: 5h
Theory classes: 1h 30m

Programming test

Description:
PC exam done during large-group hours to solve exercises of the subject by using programs in Python.

Specific objectives:
To learn the use of programming languages to resolve mathematic problems related to satellite engineering. The test will contain exercises similar to those that will have been practiced during previous course sessions.

Full-or-part-time: 6h
Self study: 5h
Theory classes: 1h

GRADING SYSTEM

Defined at the subjects' infoweb.

EXAMINATION RULES.

Tests are taken during lecture sessions, the dates of which are previously announced in ATENEA. Mid- and final-term exams are taken on the dates scheduled by EETAC.
Tests and exams must be done individually. The use of books or notes is not allowed.

BIBLIOGRAPHY

Basic:
- "Capítulo 2. Diferenciación". Marsden, Jerrold E.; Tromba, Anthony. Cálculo vectorial [on line]. Addison Wesley, 2004. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=7634.- "Capítulo 4. Funciones con valores vectoriales". Marsden, Jerrold E.; Tromba, Anthony. Cálculo vectorial [on line]. Addison Wesley, 2004. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=7634.- Larson, Ron; Edwards, Bruce H. Cálculo . 9a ed. México [etc.] : McGraw-Hill, cop. 2010. ISBN 9789701071342.

Complementary:
- Larson, Ron; Edwards, Bruce H. Cálculo . 9a ed. México [etc.] : McGraw-Hill, cop. 2010. ISBN 9788429151565.
- Barrière, Lali. Fonaments matemàtics per a l'enginyeria de telecomunicació . [Barcelona] : Edicions UPC, 2007. ISBN 9788483019078.
- Salas, Saturnino L; Hille, Einar; Etgen, Garret J. Calculus : una y varias variables . 4a ed. Barcelona [etc.] : Reverté, cop. 2002-2003. ISBN 9788429151565.

RESOURCES

Other resources:
- Course schedule with syllabus.
- Initial knowledge material.
- Slides of the contents of the course.
- List of exercises (with solutions) of the course.
- Examples of tests and exams from previous years.
- Links to notes, summaries and videos related to the concepts of the subject.

All of them are made available through ATENEA.